

Mark Saul, Ph.D.

Bronxville High School (ret.)
marksaul@earthlink.net

The number 370 is special:
 $3^3 + 7^3 + 0^3 = 27 + 343 + 0 = 370$

The number 253 is NOT special (in this way):

$2^3 + 5^3 + 3^3 = 8 + 125 + 27 = 160$,
not 253

What is the next number,
after 370, which is special
in this way?

(If you know, shhhh!
Don't tell.)



Two people play a game

There are 12 pieces.
Players take turns picking up the pieces.
The one who takes the last piece is the LOSER!
In each turn:
You must take at least ONE piece.
You may take as many as FIVE pieces.

Two people play a NEW
game

There are now **15** pieces.
Players take turns picking up the pieces.
The one who takes the last piece is the LOSER!
In each turn:
You must take at least ONE piece.
You may take as many as FIVE pieces.

Two people play a NEW
game

There are now **1000** pieces.
Players take turns picking up the pieces.
The one who takes the last piece is the LOSER!
In each turn:
You must take at least ONE piece.
You may take as many as FIVE pieces.

Two people play a NEW
game

There are now only 13 pieces.
Players take turns picking up the pieces.
The one who takes the last piece is the LOSER!
In each turn:
You must take at least ONE piece.
You may take as many as FIVE pieces.

The number 370 is special:
 $3^3 + 7^3 + 0^3 = 27 + 343 + 0 = 370$

The number 253 is NOT special (in this way):

$2^3 + 5^3 + 3^3 = 8 + 125 + 27 = 160$,
not 253

What is the next number, after 370, which is special in this way?

(If you know, shhhh! Don't tell.)



The number 370 is special:
 $3^3 + 7^3 + 0^3 = 27 + 343 + 0 = 370$

What is the next number, after 370, which is special in this way?

Answer: 371.

But are there any other such numbers?

There are five apples in a basket. How can you divide them among five people, so that each person gets an apple, and there is one apple left in the basket?



(Try this problem *mentally*)

I had some candies in my pocket. I gave half of them to my sister, then 1/3 of what was left to my brother.

I was left with 6 candies. How many did I start with?

If I was left with 6 candies after giving 1/3 of them to my brother, then the 6 candies I have left is 2/3 of what I had before that. So I must have had 9 candies before I gave my brother some.

After I gave my sister 1/2 my candies, I had 9 left. So I must have had 18 candies to begin with.

Sometimes it's useful to proceed backwards:



The Devil and the Bridge

Suppose you meet a devil, who tells you:
Here is a magic bridge.
Every time you cross the bridge, any money you have doubled.
But I will be waiting at the end of the bridge, and you have to pay me 24 cents for crossing.



You cross the bridge three times.

But after you pay the devil the third time, you have no money left.

How much money did you start with?

14

!SDRAWKCAB

There are two actions that affect the money:
crossing the bridge, and paying the devil.

If we go backwards, the devil 'gives back' 24 cents each time.

Then, going backwards over the bridge, your money is divided by two.

So:

First trip backwards: Start with 0 cents. Devil gives you 24 cents. You cross the bridge, and at the other end you have 12 cents.

Second trip backwards: Start with 12 cents. Devil gives you 24 cents, so you have 36 cents. You cross the bridge, and at the other end you have 18 cents.

Third trip backwards: Start with 18 cents. Devil gives you 24 cents, so you have 42 cents. You cross the bridge, and at the other end you have 21 cents.

Answer: 21 cents.

A, B, and C each have a certain sum of money.

For some reason, A gives half of his money to B.

Then B gives half of her money to C.

Then C gives half of his money to A.

If A now has \$33, B now has \$18, and C now has \$21, how much money did they each start with?

Solution: Working Backwards

A	B	C
33	18	21

Let A give C's "other half" back:

33-21	18	21+21
1	18	42

Next let C give B's "other half" back:

12	18+18	42-18
12	36	24

Finally, let B give A's "other half" back

12+12	36 - 12	24
24	24	24

Answer: They each had \$24 initially

There are five apples in a basket. How can you divide them among five people, so that each person gets an apple, and there is one apple left in the basket?

There are five apples in a basket. How can you divide them among five people, so that each person gets an apple, and there is one apple left in the basket?

Answer: Four people get an apple, and the fifth gets the last apple—in the basket!

There are six identical glasses in a row. The first three are full of water, and the last three are empty.

How can you arrange it so that the full and empty glasses alternate—touching only one glass?

There are six identical glasses in a row. The first three are full of water, and the last three are empty.

How can you arrange it so that the full and empty glasses alternate—touching only one glass?

ANSWER: Pour the contents of the second glass into the fifth glass.

Alice rode her bicycle south from Taipei to Tainan at a rate of 10 kilometers per hour. Betty drove her Porsche from Tainan to Taipei—along the same road—at 120 kilometers per hour.

When they met, which one was closer to Taipei?



Six young recruits stand in a single line for their first drill session. They face the gruff drill sergeant, who barks out their first order—"Left Face"—and they all react in one second this way:



Not bad - almost all of them get it right! However, each can only see the soldier in front of him. If that soldier is facing him, he assumes that he is wrong and reacts in exactly one second by turning around. After one second, the soldiers look like this:



0 faces left		1 faces right						(lookup table)					
0	1	0	1	0	0	0	1	0	1	000	0	0	000
0	0	1	0	1	0	0	0	1	1	001	1	0	001
0	0	0	1	0	1	0	0	1	1	010	2	0	001
0	0	0	0	1	0	1	0	1	1	011	3	1	011
0	0	0	0	0	1	0	1	1	1	100	4	1	010
0	0	0	0	0	0	1	1	1	1	101	5	1	011
0	0	0	0	0	0	0	1	1	1	110	6	0	101
0	0	0	0	0	0	0	1	1	1	111	7	1	111
0	0	0	0	0	0	0	1	1	1	1			
0	0	0	0	0	0	0	1	1	1	1			
0	0	0	0	0	0	0	1	1	1	1			
0	0	0	0	0	0	0	1	1	1	1			
0	0	0	0	0	0	0	1	1	1	1			
0	0	0	0	0	0	0	1	1	1	1			
0	0	0	0	0	0	0	1	1	1	1			
0	0	0	0	0	0	0	1	1	1	1			
0	0	0	0	0	0	0	1	1	1	1			

0 faces left		1 faces right						(lookup table)					
0	1	1	0	0	1	1	1	1	1	000	0	0	000
0	1	0	1	0	1	1	1	1	1	001	1	0	001
0	0	1	0	1	1	1	1	1	1	010	2	0	001
0	0	0	1	1	1	1	1	1	1	011	3	1	011
0	0	0	1	1	1	1	1	1	1	100	4	1	010
0	0	0	1	1	1	1	1	1	1	101	5	1	011
0	0	0	1	1	1	1	1	1	1	110	6	0	101
0	0	0	1	1	1	1	1	1	1	111	7	1	111
0	0	0	1	1	1	1	1	1	1	1			
0	0	0	1	1	1	1	1	1	1	1			
0	0	0	1	1	1	1	1	1	1	1			
0	0	0	1	1	1	1	1	1	1	1			
0	0	0	1	1	1	1	1	1	1	1			
0	0	0	1	1	1	1	1	1	1	1			
0	0	0	1	1	1	1	1	1	1	1			
0	0	0	1	1	1	1	1	1	1	1			
0	0	0	1	1	1	1	1	1	1	1			

Alice rode her bicycle south from Taipei to Tainan at a rate of 10 kilometers per hour. Betty drove her Porsche from Tainan to Taipei—along the same road—at 120 kilometers per hour.

When they met, which one was closer to Taipei?



Alice rode her bicycle south from Taipei to Tainan at a rate of 10 kilometers per hour. Betty drove her Porsche from Tainan—along the same road—at 120 kilometers per hour.

When they met, which one was closer to Taipei?

Answer: When they met, they were at the same place!

So they were both the same distance from Taipei, or from anywhere else.

Isn't that silly?

